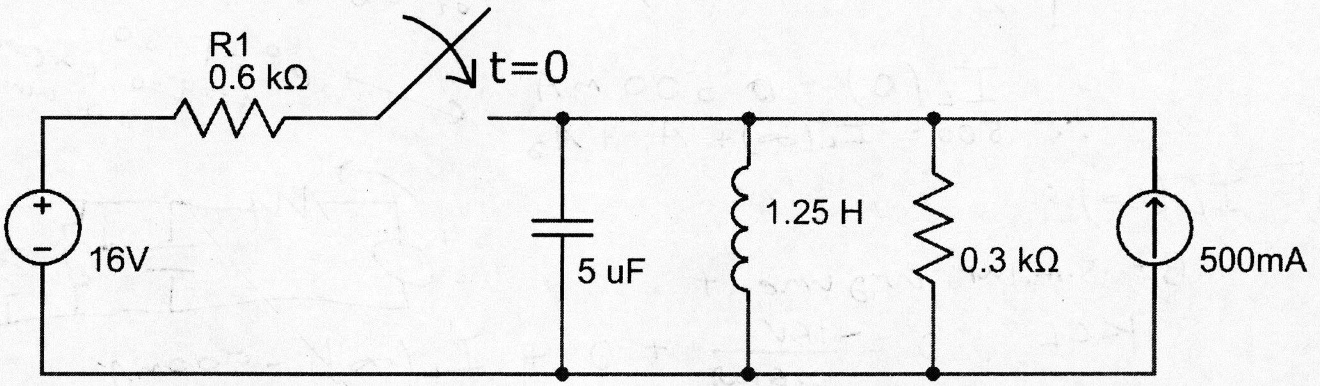
ECE 2001-002D : Jeffrey Santi　　　Quiz 3　　　Version A

Find the current IL(t) through the inductor and voltage VC(t) across the capacitor for t > 0. Be sure to indicate your polarity and current direction.

Some Helpful Equations: $\omega_o = 1/\text{sqrt}(LC)$, $\alpha = 1/(2RC)$ for parallel, $\alpha = R/(2L)$ for series
$A_1e^{s1t} + A_2e^{s2t}$
$D_1e^{-\alpha t} + tD_2e^{-\alpha t}$
$B_1e^{-\alpha t}\cos(\omega_d t) + B_2e^{-\alpha t}\sin(\omega_d t)$



① Find α, w_o

- Well, $R_{Th} = 0.6k\Omega \,||\, 0.3k\Omega = 0.2k\Omega$
- $W_o = \frac{1}{\sqrt{LC}} = 400$ Rad/s　　$\alpha = \frac{1}{2RC} = 500$ Rad/s

② Find General Solution

- $S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - w_o^2} = \{-800, -200\}$
- Since we have real Solutions (unique ones)

Overdamped $\therefore$ $x(t) = x(\infty) + A_1e^{s_1t} + A_2e^{s_2t}$

- In the parallel case:
  - $I_c(t)$ is discontinuous
  - $V(0) = 0$ and $V(\infty) = 0$ so we couldn't find the constants.
  - This leaves us with $I_L(t)$

Note

$V_c = V_L$

- $I_L(t) = I(\infty) + A_1e^{-200t} + A_2e^{-800t}$

③ Find constants

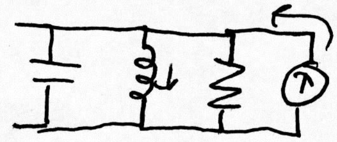
We can Find $I_L(\infty)$ and $I_L(0)$ using circuit analysis

A third equation is needed:

Note $V_L = L\frac{dI_L}{dt}$ and that $V_L = V_c$ and V_c is continuous

So we use $V_L(0)$

a) $I_L(0)$:



Since circuit settled $\frac{dI_L}{dt} = 0$

thus $V_L(0) = 0$

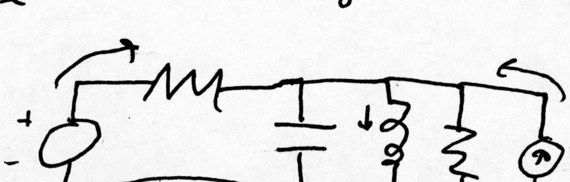
By KCL : $0 = 0 + I_L(t) + 0 - 500mA$

| 1) Ohm's Law shows resistor current is zero
| 2) Since settled, $I_c = \frac{dV_c}{dt} = 0$

$\therefore I_L(0) = 500 mA$

$\therefore 500 = I_L(\infty) + A_1 + A_2$ ← $e^0 = 1$ so At $t=0$, exponentials go away

b) $I_L(\infty)$:



By similar argument

KCL : $0 = \frac{-16V}{0.6k\Omega} + \underbrace{0}_{I_c(\infty)} + I_L(\infty) - 500mA$

$\therefore I_L(\infty) = 526.7 \text{ mA}$

c) Since settled $\frac{dI_L}{dt} = 0 \therefore V_L(0) = 0$

$$0 = L\frac{d}{dt}\left[I(\infty) + A_1e^{-200t} + A_2e^{-800t}\right]\Big|_{t=0}$$

$$\frac{0}{L} = -200A_1e^0 + -800A_2e^0$$

$$0 = -200A_1 - 800A_2$$

d) $\begin{cases} I_F = 526.7 \\ -200A_1 - 800A_2 = 0 \\ I_F + A_1 + A_2 = 500 \end{cases}$ $\therefore$ $I_F = 526.7 mA$, $A_1 = -35.6 mA$, $A_2 = 8.9 mA$

(5) Specific Solution

$$I_L(t) = \left[526.7 + 8.9e^{-800t} - 35.6e^{-200t}\right] mA$$

(6) Finding $V_c(t)$

Note $V_c = V_L$ ← In parallel and $V_L = L\frac{dI_L}{dt}$

$$V_c(t) = 1.25H\left[(-800)(.0089A)e^{-800t} + (200)(0.0356A)e^{-200t}\right]$$

$$= \left[8.9e^{-200t} - 8.9e^{-800t}\right] V$$

RLC General Tips

As Before, these are critical:

$$\boxed{V_L = L\frac{dI_L}{dt}} \qquad \boxed{I_C = C\frac{dV_C}{dt}}$$

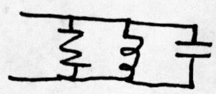
From this we know $I_L(t)$ and $V_C(t)$ must be continuous, otherwise we get an undefined or infinite voltage/current

Additionally, the general solution to the second order circuit is:

$$\begin{cases} x(t) = x(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t} & [OD] \\ x(t) = x(\infty) + D_1 e^{-\alpha t} + D_2 t e^{-\alpha t} & [CD] \\ x(t) = x(\infty) + B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t) & [UD] \end{cases}$$

So which variable is $x(t)$?

Parallel



$V_L = V_C$

Unless Given $V(0)$:

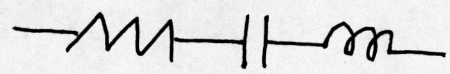
- Circuit probably settled $\frac{d\ldots}{dt} = 0$

  so $I_C(0^-) = 0$, $V(0) = 0$
  $I_C(\infty) = 0$, $V(\infty) = 0$

- The only way to find the constants is for $x(t) \rightarrow \boxed{I_L(t)}$

Use $L\frac{dI_L}{dt}$, $I_L(0)$, $I_L(\infty)$ to find constants

Series



$I_L = I_C$

Unless Given $I(0)$:

- Circuit probably settled $\frac{d\ldots}{dt} = 0$

  so $V_L(0^-) = 0$, $I(0) = 0$
  $V_L(\infty) = 0$, $I(\infty) = 0$

- The only way to find the constants is for $x(t) \rightarrow \boxed{V_C(t)}$

Use $C\frac{dV_C}{dt}$, $V_C(0)$, $V_C(\infty)$ to find constants