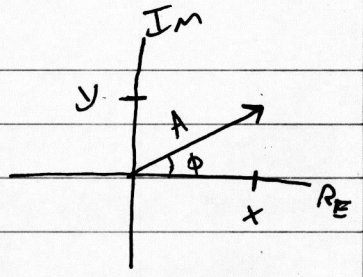
Complex Numbers

$x + jy$ where x is real component
y is imag component



Polar Form $A \angle \phi = A e^{j\phi}$

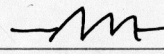
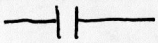
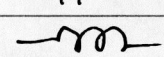
Euler's Formula Links these forms

$Ae^{j\phi} = A\cos\phi + jA\sin\phi$

Conversion: $A = \sqrt{x^2+y^2}$ $\phi = \tan^{-1}\left(\frac{y}{a}\right)$

General AC Steady State Steps

1) Convert components to phasor form

	$R \rightarrow R$
	$C \rightarrow \frac{-j}{\omega C}$
	$L \rightarrow j\omega L$

$A\cos(\omega t + \phi) \rightarrow A \angle \phi$

$A\sin(\omega t + \phi) \rightarrow A \angle \phi - 90°$

2) Solve Circuit using phasor form

3) Convert back to time domain

$A \angle \phi \rightarrow A\cos(\omega t + \phi)$

$\omega = 1000$, so

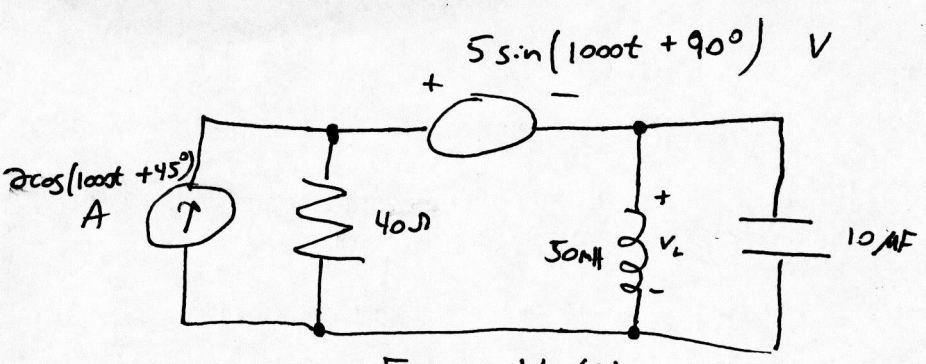
$2\cos(1000t + 45°) \rightarrow 2\angle 45°$ A

$40\Omega \rightarrow 40\Omega$

$5\sin(1000t + 90°) \rightarrow 5\angle 0°$ V

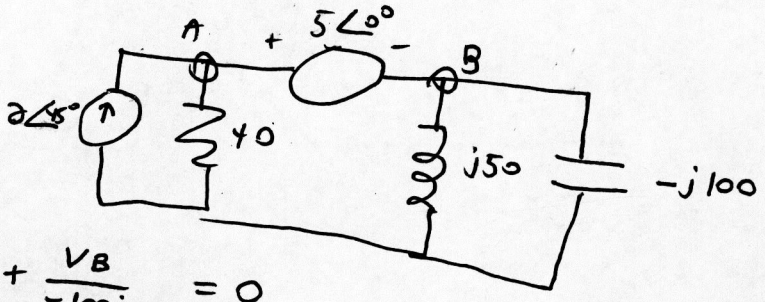
$50mH \rightarrow j50$

$10\mu F \rightarrow -100j$



Find $V_L(t)$

Node Voltage Method:



A and B form a supernode

SN $\quad -2\angle 45° \text{ A} + \frac{V_A}{40\Omega} + \frac{V_B}{j50} + \frac{V_B}{-100j} = 0$

$-2\angle 45° + \frac{V_A}{40} + V_B\left[\frac{-j}{50} + \frac{j}{100}\right] = 0$

NUM $\quad -V_A + 5 + V_B = 0$

$V_B \cdot \left[\frac{-j}{100}\right] = 2\angle 45° \cdot -\frac{V_A}{40}$

$V_A = 40\left[2\angle 45° + \frac{j}{100} V_B\right]$

Note that $V_A = V_B + 5$ from NUM

$V_B + 5 = 40\left[2\cos 45° + 2j\sin 45° + \frac{j}{100} V_B\right]$

$V_B = -5 + 56.6 + j56.6 + j0.4 V_B$

$V_B[1 - 0.4j] = 51.6 + j56.6$

$V_B[1.08\angle -21.8°] = 76.6\angle 47.6°$

$\therefore V_B = \frac{76.6}{1.08}\angle 47.6° - (-21.8°) = 70.9\angle 69.4°$

$V_B(t) = 70.9\cos(1000t + 69.4°)$